# ANALOG COMPUTER STUDIES OF FREQUENCY MULTIPLICATION AND MIXING WITH THE JOSEPHSON JUNCTION

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## ABSTRACT

Using a point-contact Josephson junction (JJ), direct frequency measurement of far-IR laser lines can be performed by mixing the Nth harmonic of a microwave frequency $\nu_2$ with the laser frequency $\nu_1$ to produce a beat signal $\nu_{IF}$ such that $\nu_{IF} = \nu_1 - N\nu_2$. Analog computer simulation of the JJ has revealed an efficient mode of frequency multiplication and mixing. This is a condition wherein the self oscillation, $\nu_J$, is phase locked to a frequency $\nu_J = \pm \ell\nu_2 \pm k\nu_1 \pm m\nu_{IF}$ where $\ell$, k, and m are integers. The analog studies show that this phase locking can occur at very low as well as at high levels of the external drives. The result of the phase lock is an efficient transfer of energy into the $\nu_{IF}$ output signal. At least one experimental result has verified the occurrence of phase locking to difference frequencies. It is also well known that the optimum bias points in mixing lie between the $\nu_2$ steps. The interpretation of these results and the direct role played by $\nu_J$ in mixing experiments has not, however, been generally recognized.

## I. INTRODUCTION AND THE MODEL

For frequency multiplication from microwave frequencies into the far infrared, the point contact Josephson junction (JJ) is without peer. With this device, multiplication by 825 has been achieved and a laser frequency of 3.8 THz has been measured [1,2].The procedure is to mix the Nth harmonic of a microwave signal with the laser fundamental to produce a beat of frequency

$$\nu_{IF} = \nu_1 - N\nu_2 \tag{1}$$

where $\nu_1$ and $\nu_2$ are the laser and microwave frequencies respectively. However, for large values of N (say 200 or more), the signal at $\nu_{IF}$ is small at best and the probability of a useable signal has been poor.

To understand the effects of various parameters we have performed analog computer [3] studies of the mixing and harmonic generation process. This work has shown the importance of a mode of operation wherein the self oscillation at frequency $\nu_J$, is at least intermittently phase locked to the frequency

$$\nu_J = \pm \ell\nu_2 \pm k\nu_1 \pm m\nu_{IF} \tag{2}$$

where $\ell$, k, and m are integers. Our claim of phase locking is based on the familiar "constant voltage" steps in the $\overline{I}$ versus $\overline{V}$ curve of the JJ [4]. (The time-averaged current through the junction is $\overline{I}$ and the time-averaged voltage across it is $\overline{V}$).In the computer study - in addition to the steps due to $\nu_1$, $\nu_2$ and their harmonics - there are steps at values of $\overline{V}$ given by $\overline{V} = \nu_J(h/2e)$, where $\nu_J$ is given by Equation 2.

The model for a low-capacitance point-contact junction is assumed to be the simple resistively shunted JJ as shown in Fig. 1. That portion of the JJ representing the supercurrent, $I_J$, is given by the usual two equations $I_J = I_c \sin\phi$; $\frac{d\phi}{dt} = \frac{2e\,V}{\hbar}$. Since the junction normal resistance is generally low compared to both the dc bias impedance and the microwave and laser impedances we approximate the drive as a constant current source. The results obtained using this constant current model are generally in much better agreement with real data for point-contact junctions than the results obtained using a constant voltage model. For example, the absolute power of the signal at $\nu_{IF}$ and saturation levels are better predicted by the constant current model. (Throughout this paper, the symbols $I_1$ and $I_2$ represent the laser and microwave drive currents respectively).

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## II. BASIC RESULT OF STUDY

Figure 2 represents the essence of our work and it will now be discussed in some detail. In this figure, peaks in $I_{J_{IF}}$ ($I_{J_{IF}}$ is the spectral component of the Josephson current at the frequency $\nu_{IF}$) and steps in $\overline{I}$ are plotted versus $\overline{V}/V_{Ref}$. Throughout the paper, $V_{Ref} \equiv \nu_2(h/2e)$ and $\nu_c \equiv I_c R_s(2e/h)$ where $I_c$ and $R_s$ are the junction critical current and shunt resistance respectively. The normalization factor, $I_{Ref}$ is that value of $I_1$ which produces the first maximum of $J_1(\beta_1)$. Here $\beta_1$ is that value of $I_1$ which produces the first maximum of $J_1(\beta_1)$. Here $\beta_1$ is the argument of the Bessel function of first order and zero-th kind. It is given by:

$$\beta_1 = \frac{2e\,I_1\,R_s}{\hbar\,\omega_1}$$

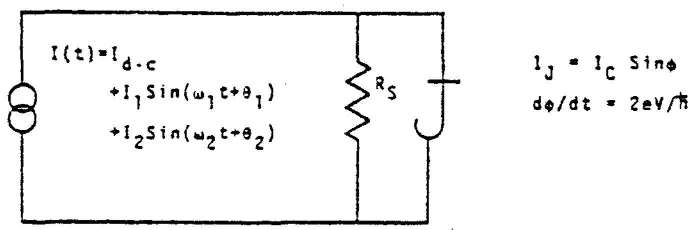


Fig. 1. Schematic of resistively shunted Josephson junction driven by constant-current sources. $I_1$ is the laser drive current and $I_2$ is the microwave drive current.

The usual steps due to the microwave and laser sources are given, respectively, by terms one and two of Equation 2. The third term - in combination with one or the other of the first two terms - describes steps that are due to the mixing of the fundamental of the laser with the Nth harmonic of the microwave source. That is, steps involving $\nu_{IF} = \nu_1 - N\nu_2$. The equation, $\nu_J = \ell\nu_2 + k\nu_1$ (with the indices $\ell$ and k taking all positive and negative integer values) defines all the possible frequencies resulting from a linear combination of the drives at $\nu_2$ and $\nu_1$. Equation 2 can, of course, be reduced to this form. We have written Equation 2 simply to conceptualize the fact that the individual drives produce lines at the same <u>frequencies</u> as they would if applied separately. The <u>simultaneous</u> application of these two drives results in the original lines (although they may be radically altered in amplitude and phase) as well as additional

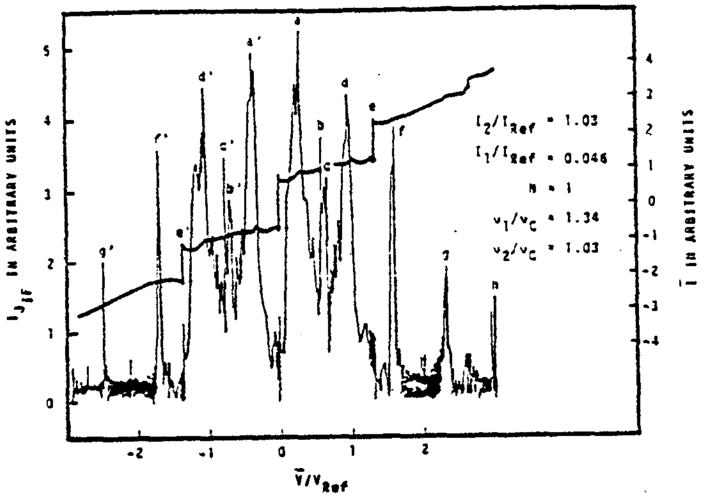


Fig. 2. Analog simulation for N = 1. $I_{J_{IF}}$ and $\bar{I}$ plotted simultaneously against $\bar{V}/V_{Ref}$. $\bar{V}_{Ref} \equiv \nu_2(h/2e)$. Both vertical axes are in arbitrary units. $I_{Ref}$ is defined in text. The following list defines the frequencies (actually, the values of $\bar{V}$) associated with each of the letters in the figure: $a = \nu_{IF}$, $a' = -\nu_{IF}$, $b = 2\nu_{IF}$, $b' = -2\nu_{IF}$, $c = \nu_2 - \nu_{IF}$, $c' = -\nu_2 + \nu_{IF}$, $d = \nu_2$ and $\nu_1 - \nu_{IF}$, $d' = -\nu_2$ and $-\nu_1 + \nu_{IF}$. $e = \nu_1$ and $\nu_2 + \nu_{IF}$, $e' = -\nu_1$ and $-\nu_2 - \nu_{IF}$. $f = \nu_1 + \nu_{IF}$ and $\nu_2 + 2\nu_{IF}$, $f' = -\nu_1 - \nu_{IF}$ and $-\nu_2 - 2\nu_{IF}$, $g = 2\nu_2 + \nu_{IF}$ and $2\nu_1 - \nu_{IF}$, $g' = -2\nu_2 - \nu_{IF}$ and $-2\nu_1 + \nu_{IF}$, $h = 2\nu_1 + \nu_{IF}$.

lines. One final point. There are, of course, no negative frequencies. All the lines at negative values of $\bar{V}/V_{Ref}$ merely represent the fact that phase locking can - and does - occur at negative values of $\bar{V}$. In Fig. 2 the major peaks of $I_{J_{IF}}$ have been identified. For example, the peak at $\bar{V}/V_{Ref}$ corresponding to the frequency $2\nu_1 + \nu_{IF}$ (line h) is obtained with $\ell = 0$, $k = 2$ (and plus sign), and $m = 1$ (and plus sign) in Equation 2. It can also be seen that degeneracies are a possibility such as $\nu_1 - \nu_{IF}$ and $\nu_2$ (line d).

One aspect of this degeneracy is that the $I_{J_{IF}}$ peaks can be quite small where one might expect them to be fairly large. For example, the two largest steps in the $\bar{I} - \bar{V}$ curve (other than the zero-th step), which occur at $\bar{V}/V_{Ref}$ of about - 1.3 and + 1.3, correspond to lines e$'$($-\nu_1$, $-\nu_2 - \nu_{IF}$) and e ($+\nu_1$, $+\nu_2 + \nu_{IF}$) respectively. The IF peaks here are at least weak if not missing. On the other hand, the degeneracies labeled lines d$'$( $-\nu_2$, $-\nu_1 + \nu_{IF}$ and d ($\nu_2$, $\nu_1 - \nu_{IF}$) correspond to large IF peaks. These two cases appear to be explained by noting that in the case of the missing peaks the steps in the $\bar{I} - \bar{V}$ curve are due almost entirely to the rather large drive at $\nu_1$ ($I_1/I_{Ref}$ = 0.34). We interpret this to mean that most of the spectral content of $I_J$ is confined to $\nu_1$ and its harmonics. In the other case, the terms having to do with just the external drive ($-\nu_2$, $\nu_2$) are weak and the other terms ($-\nu_1 + \nu_{IF}$ and $\nu_1 - \bar{\nu}_{IF}$) result in a significant part of the supercurrent appearing in the $\nu_{IF}$ component. In other analog data (not shown) we have seen that these two cases are reversed by changing the relative sizes of the two drives.

This interpretation of the relative sizes of $I_{J_{IF}}$ peaks in the presence of degeneracy is consistent with points 1 and 2 of the low-temperature data in Section III. With respect to this, it should be noted that for most data $\nu_1/\nu_2 > 3$ so that the degeneracies (of low-order) that caused the missing peaks in Fig. 2 don't occur. It should also be noted that, despite the complexity of Fig. 2, the numerical values of $\ell$, k, and m are always small - never more than 2. That is, low-order processes dominate as one would expect. A study of additional analog data (not shown) indicates that the relative heights of the various peaks are strongly influenced by the relative sizes of the two ac drives.

## III. COMPARISON OF COMPUTER SIMULATION AND LOW-TEMPERATURE DATA

### Computer Results

There are five basic features of the computer simulation:

(1) Peaks in the intermediate frequency power, $P_{IF}$, occur at values of $\bar{V}$ corresponding to the frequencies $\nu_J = \pm \ell\nu_2 \pm k\nu_1 \pm m\nu_{IF}$, except as discussed in Section II.

(2) Nulls in $P_{IF}$ occur for $\bar{V}$ values corresponding to $\nu_J = \ell\nu_2$ and $\nu_J = k\nu_1$, except as discussed in Section II.

(3) $P_{IF}$ saturates for laser power levels considerably less than predicted by the constant-voltage model.

(4) $P_{IF}$ increases linearly with $P_{laser}$ for small values of $P_{laser}$.

(5) As N increases, the peaks in $P_{IF}$ versus $\bar{V}$ - in between an adjacent pair of microwave steps - become smaller. The background level between these peaks increases with N and the resolution of the individual peaks becomes more difficult.

### Low-Temperature Point-Contact JJ Results

The data on frequency multiplication with JJ's fall roughly into two categories: first, N values in the range 1 to 12 [5,6,7,8]. Second, N's from about 50 to 825 [1,2,9,10].

In the second category, information about the dependence of $P_{IF}$ on various parameters is imprecise but both categories support all the points below.

(1) $P_{IF}$ maximizes at $\bar{V}$ values between those corresponding to the microwave steps, i.e., for $\nu_J$ between $\ell\nu_2$ and $(\ell + 1)\ \nu_2$ [5,6,7,8,10].

(2) Nulls in $P_{IF}$ occur for $\bar{V}$ values corresponding to $\nu_J = \ell\nu_2$ [5,6].

(3) Saturation of $P_{IF}$ occurs for $P_{laser}$ greater than about 0.1 $P_{\mu\omega}$ for values of N of the order of 10 and less [7]. For larger N the two drives need to be more nearly equal [1]. $P_{\mu\omega}$ is the microwave power and is roughly

optimum when it maximizes the Nth microwave step in the $\bar{I}$ - $\bar{V}$ curve [7].

(4) $P_{IF}$ increases linearly with $P_{laser}$ for $P_{Laser} << 0.1\ P_{\mu\omega}$ [6,7].

(5) For N values beyond about 10 there is only one broad maximum in $P_{IF}$ between microwave steps [7].

Reference 7 (N = 9 and 12) found that there was an optimum bias point between any one pair of steps, that there was a best bias point, and that the harmonic number influences between which pair of steps it occurs. Furthermore, for steps beyond the 6th, the size of the maxima decreased significantly. The analog results show the same qualitative dependence upon harmonic number.

The phase locking interpretation is strengthened by one instance of data wherein steps appeared in the $\bar{I}$ - $\bar{V}$ curve in accordance with Equation 2. In Reference 5, a point-contact JJ was irradiated with a 64 GHz and a 72 GHz klystron. Steps in the $\bar{I}$ - $\bar{V}$ curve (their Fig. 4) clearly appear in positions at least qualitatively in agreement with Equation 2. Steps corresponding to m values as high as 2 are visible. Unfortunately, no $P_{IF}$ data with $\nu_{IF}$ = 8 GHz were taken.

## IV. FURTHER DETAILS OF ANALOG RESULTS

Data were taken of $I_{J_{IF}}$ versus $\bar{V}$ with $I_1$ and N as parameters. The N value studied were 1,3, and 10. Practical limitations of the analog simulator prevented useful analysis for N values much above 10. For N values of 1 and 3 the biggest maximum in $I_{J_{IF}}$ occurs for $\ell$ = 0, k = 0, m = + 1 (i.e., $\nu_J = \nu_{IF}$) in Equation 2. The data discussed below were taken under that condition. In this section the following conditions hold: N = 1; $\nu_1/\nu_c$ = 3.38, $\nu_2/\nu_c$ = 3.10. N = 3; $\nu_1/\nu_c$ = 3.38, $\nu_2/\nu_c$ = 1.03. N = 10; $\nu_1/\nu_c$ = 3.23, $\nu_2/\nu_c$ = 0.310. For all cases, the normalized bandwidth of the spectrum analyzer was $1 \times 10^{-3}$.

For N = 1 we consistently observed that there was a range of $\bar{V}$ over which $I_{J_{IF}}$ exhibited a smooth peak with a single maximum. (The data of Fig. 2 suffer from the fact that the $\bar{V}$ range was swept too fast for a complete response). Outside this range - but still between $\ell$ = 0 and $\ell$ = 1 - the output of the spectrum analyzer was rather unstable. Inside the range it was quite stable.

For each value of N and $I_1$ there is a maximum in $I_{J_{IF}}$ within a range of $\bar{V}$ similar to that discussed above. Figure 3 shows these maxima versus $I_1$.

This brings us to another basic point. Although, the output we wish to detect is at $\nu_{IF}$, energy is contained - often of significant magnitude - in many other spectral lines. In general, there are spectral components at the fundamentals and harmonics of each of the three frequencies $\nu_{IF}$, $\nu_1$ and $\nu_2$. In addition, each of these lines produce sum and difference frequencies with each of the other lines. Operating in the phase-locked mode we have examined this spectrum versus $I_1$, $I_2$, and N. The lines of significant strength are restricted to low orders of harmonic generation and mixing. Nevertheless, the spectrum has a very large number of intense lines. Fig. 4 shows a typical spectrum when phase locked. Here, $\nu_J = \nu_{IF}$, $I_2/I_{Ref}$ = 1.0 and $I_1/I_{Ref}$ = 0.23.

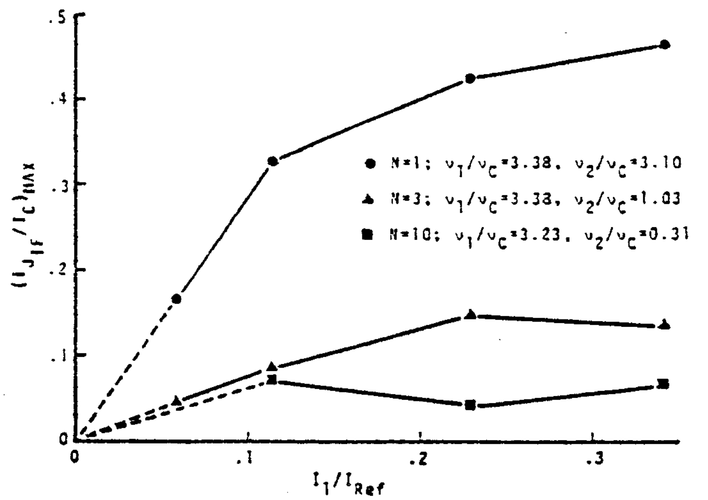


Fig. 3. The maximum values of $I_{J_{IF}}$ versus $I_1$ with N as parameter.

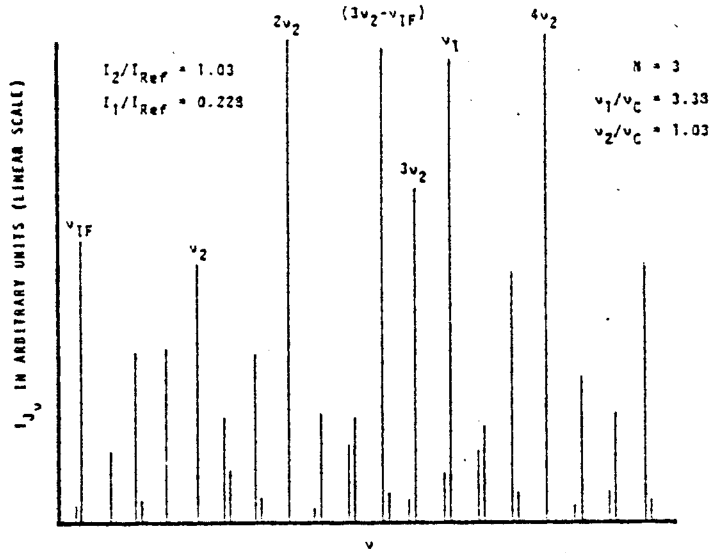


Fig. 4. The major lines in the spectrum of the Josephs current. Spectrum taken under phase-locked condiction with $\nu_J = \nu_{IF}$. Here N = 3, and lines shown as delta functions for simplicity.

What determines whether the various peaks in a plot such as Fig. 2 can be resolved? Two factors: One is the N value and the other is the noise of the ac sources and the dc bias. The N value is important because, as N increases, the spectrum spreads out and the fraction of the supercurrent that goes into $I_{J_{IF}}$ decreases. If the ac drives and the dc bias are noisy this will adversely effect the phase lock [11]. Noise in the lower frequency drive ($\nu_2$) is particularly determintal because the degradation increases rapidly with increasing N[12].

## V. THE NEED FOR A SIMPLIFIED ANALYSIS

The computer simulations discussed above will not permit useful predictions at sufficiently high values of N. What is needed is a simplfied analysis that will predict $I_{J_{IF}}$ versus N and $I_1$ for just one value of $\bar{V}$ (say that corresponding to $\nu_J = \nu_{IF}$). We know of no such analysis.

We have made one attempt at such a calculation. Here we used a self-consistent calculation with $\phi$( the $\phi$ in $I_J = I_c \sin\phi$) expanded in four terms. These had frequency dependences of dc, $\nu_1$, $\nu_2$, and $\nu_{IF}$. This calculation showed phase-locking features such as steps in the $\bar{I}$ - $\bar{V}$ curve according to Equation 2. It did not, however, give useful estimates (i.e., in agreement with the simulations) of $I_{J_{IF}}$ even for N = 1. Perhaps the assumptions of this calculation contrasted with the data of Figure 4 will point the way to a better approach.

## VI. CONCLUSIONS

The analog simulation shows that the phase locked mode of operation is quite important at low values of N. The limited data available with actual JJ's at low N suggests that this is, in fact, how the JJ's have been operated. Since, with the analog simulator, no other efficient mode of operation has been found, we presume that phase locking - at least intermittently - applies even at large N values. To increase the efficiency of the phase locking process the noise on the ac and dc drives should be minimized. In addition, it may be possible to resonate the external circuitry in which the JJ is placed so that the spectrum of $I_J$ is modified. Modifications may be possible which result in forcing more energy into $I_{J_{IF}}$.

## REFERENCES


[1] T. G. Blaney and D. J. E. Knight, "Direct 825th harmonic mixing of a 1 GHz source with an HCN laser in a Josephson junction," J. Phys. D: Appl. Phys., Vol. 7, pp. 1882-1886, 1974.

[2] D. G. McDonald, A. S. Risley, J.D. Cupp, K. M. Evenson, and J. R. Ashley, "Four hundredth-order harmonic mixing of microwave and infrared laser radiation using a Josephson junction and a maser," Appl. Phys. Lett., Vol. 20, pp. 296-299, Apr. 1972.

[3] C. A. Hamilton, "Analog simulation of a Josephson junction," Rev. Sci. Inst. 43, No. 3, Mar. 1972.

[4] D. N. Langenberg, D. J. Scalapino, and B. N. Taylor, "Josephson-type superconducting tunnel junctions as generators of microwave and submillimeter wave radiation," Proc. IEEE, Vol. 54, No. 4, Apr. 1966.

[5] C. C. Grimes and Sidney Shapiro, "Millimeter-wave mixing with Josephson junctions," Phys. Rev., Vol. 169, No. 2, pp. 397-406, May 1968.

[6] T. G. Blaney, "Heterodyne laser radiation detection at 891 GHz using Josephson point contacts," Rev. Phys. Appl., Vol. 9, pp. 279-284, 1974.

[7] T. G. Blaney and D. J. E. Knight, "Heterodyne reception at 891 GHz using a Josephson harmonic mixer and a microwave local oscillator," J. Phys. D: Appl. Phys., Vol. 7, pp. 1887-1893, 1974.

[8] Y. Taur, J. H. Claassen, and P. L. Richards, "Josephson junctions as heterodyne detectors," IEEE Trans. MTT, Vol. 22, No. 12, pp. 1005-1009, Dec. 1974.

[9] D. G. McDonald, A. S. Risley, J. D. Cupp and K. M. Evenson, "Harmonic mixing of microwave and far-infrared laser radiation using a Josephson junction," Appl. Phys. Lett., Vol. 18, No. 4, pp. 162-164, Feb. 1971.

[10] T. G. Blaney and D. J. E. Knight, "Experiments using a superconducting point-contact harmonic mixer near 1 THz," J. Phys. D: Appl. Phys., Vol. 6, pp. 936-952, 1973.

[11] V. E. Kose and D. B. Sullivan, "Influence of external noise on microwave-induced Josephson steps," J. Appl. Phys., Vol. 41, No. 1, pp. 169-174, Jan. 1970.

[12] F. L. Walls and A. DeMarchi, "RF spectrum of a signal after frequency multiplication; measurement and comparison with a simple calculation," IEEE Trans. I&M, Vol. 24, No. 3, pp. 210-217, 1975.